In Support Vector Machines (SVMs), the result of the kernel function plays a crucial role in the computation of the decision boundary or hyperplane. Here’s how the SVM uses the kernel function result:

1. \*\*Computing Pairwise Similarities:\*\*

The kernel function \( K(x\_i, x\_j) \) measures the similarity between two data points \( x\_i \) and \( x\_j \). In a higher-dimensional space, this is analogous to computing the dot product of these data points. The kernel function provides this similarity without explicitly transforming the data into that higher-dimensional space.

### 2. \*\*Formulating the Optimization Problem:\*\*

SVMs aim to find the hyperplane that best separates the classes. The optimization problem in SVMs involves finding this hyperplane by maximizing the margin between classes. This problem can be formulated using Lagrange multipliers, which leads to a dual problem where the kernel function comes into play.

The dual problem is expressed in terms of the kernel function and involves maximizing an objective function subject to constraints. The objective function is:

\[ \text{maximize} \sum\_{i=1}^n \sum\_{j=1}^n \alpha\_i \alpha\_j y\_i y\_j K(x\_i, x\_j) \]

where \( \alpha\_i \) are the Lagrange multipliers, \( y\_i \) are the class labels, and \( K(x\_i, x\_j) \) is the kernel function.

### 3. \*\*Determining Support Vectors:\*\*

Once the optimization problem is solved, the resulting Lagrange multipliers \( \alpha\_i \) indicate which data points are support vectors (those with non-zero multipliers). The kernel function helps in determining the contribution of these support vectors to the decision boundary.

### 4. \*\*Constructing the Decision Function:\*\*

The decision function or classifier can be expressed using the support vectors and their associated Lagrange multipliers:

\[ f(x) = \text{sgn} \left( \sum\_{i=1}^n \alpha\_i y\_i K(x\_i, x) + b \right) \]

Here, \( x \) is a new data point for which we want to make a prediction, and \( b \) is the bias term.

### 5. \*\*Making Predictions:\*\*

To classify a new data point, the SVM uses the decision function to determine on which side of the hyperplane the point lies. The result of the kernel function \( K(x\_i, x) \) helps compute this decision value based on the similarities between the new data point and the support vectors.

### Summary

- \*\*Kernel Function\*\*: Computes the similarity between pairs of data points in the original space.

- \*\*Optimization\*\*: The kernel function is used in the dual formulation of the SVM’s optimization problem.

- \*\*Support Vectors\*\*: Determine which data points are critical for defining the decision boundary.

- \*\*Decision Function\*\*: The result of the kernel function is used to construct and evaluate the decision function for classifying new data points.

The kernel function allows SVMs to handle non-linear classification problems effectively by implicitly working in a higher-dimensional space where the data might be linearly separable.